Digital Image Processing

Lecture 11. Image Segmentation

Autumn 2010

Fundamentals

- Let R represent the entire spatial region occupied by an image. Image segmentation is a process that partitions R into *n* sub-regions, R_1, R_2, …, R_n, such that

(a) $\bigcup_{i=1}^{n} R_i = R$.

(b) R_i is a connected set. $i = 1,\ 2,\ \ldots,\ n$.

(c) $R_i \cap R_j = \Phi$.

Image Segmentation

- Segmentation is to subdivide an image into its component regions or objects.
- Segmentation should stop when the objects of interest in an application have been isolated.
- Segmentation algorithms generally are based on one of 2 basis properties of intensity values
  - discontinuity : to partition an image based on sharp changes in intensity (such as edges)
  - similarity : to partition an image into regions that are similar according to a set
    of predefined criteria.

Detection of Discontinuities

- detect the three basic types of gray-level discontinuities
  - points , lines , edges
- the common way is to run a mask through the image

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Background

- First-order derivative

$$\frac{\partial f}{\partial x} = f'(x) = f(x+1) - f(x)$$

- Second-order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

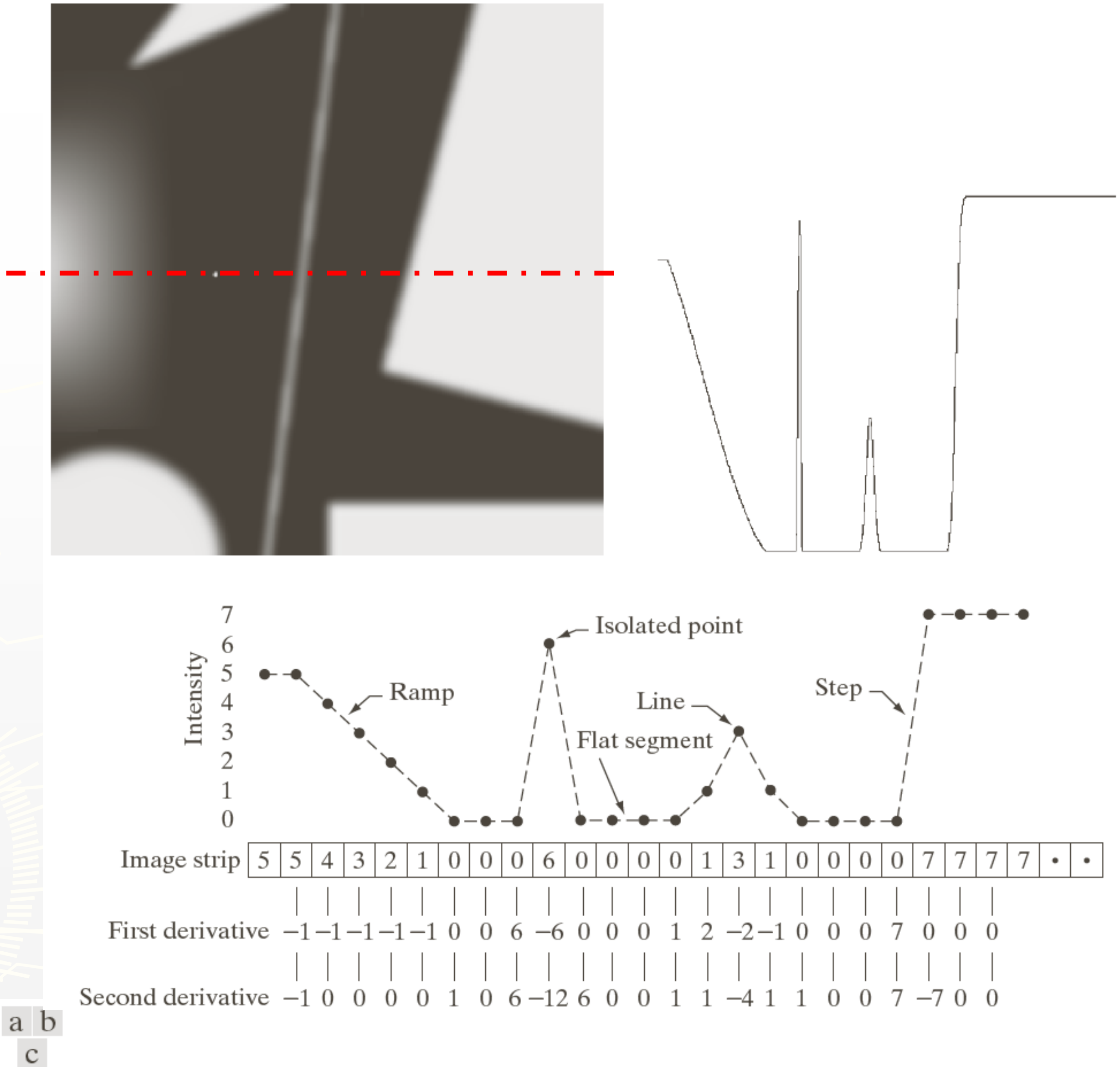


FIGURE 10.2 (a) Image. (b) Horizontal intensity profile through the center of the image, including the isolated noise point. (c) Simplified profile (the points are joined by dashes for clarity). The image strip corresponds to the intensity profile, and the numbers in the boxes are the intensity values of the dots shown in the profile. The derivatives were obtained using Eqs. (10.2-1) and (10.2-2).

Characteristics of First and Second Order Derivatives

- First-order derivatives generally produce thicker edges in image
- Second-order derivatives have a stronger response to fine detail, such as thin lines, isolated points, and noise
- Second-order derivatives produce a double-edge response at ramp and step transition in intensity
- The sign of the second derivative can be used to determine whether a transition into an edge is from light to dark or dark to light

Point Detection

- a point has been detected at the location on which the mask is centered if

$$|R| \geq T$$

- where
  - T is a nonnegative threshold
  - R is the sum of products of the coefficients with the gray levels contained in the region encompassed by the mask.

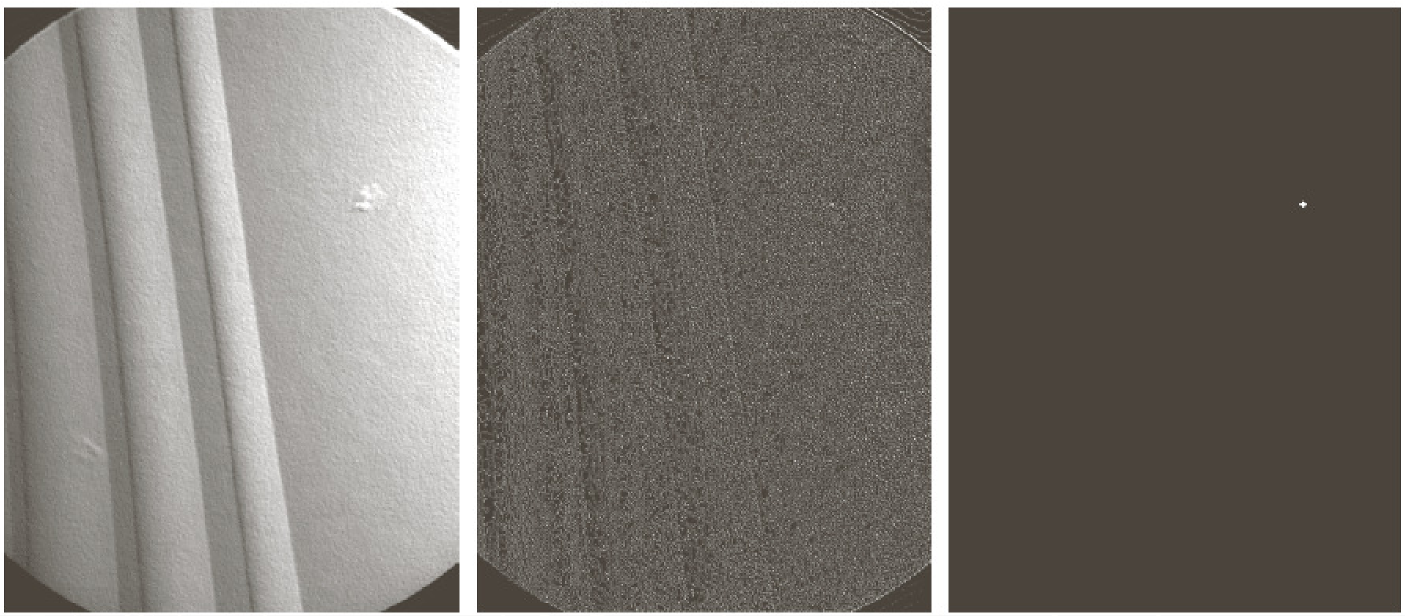
−1	−1	−1
−1	8	−1
−1	−1	−1

Point Detection

- Note that the mask is the same as the mask of Laplacian Operation (in chapter 3)
- The only differences that are considered of interest are those large enough (as determined by T) to be considered isolated points.

$$|R| \geq T$$

1	1	1
1	−8	1
1	1	1



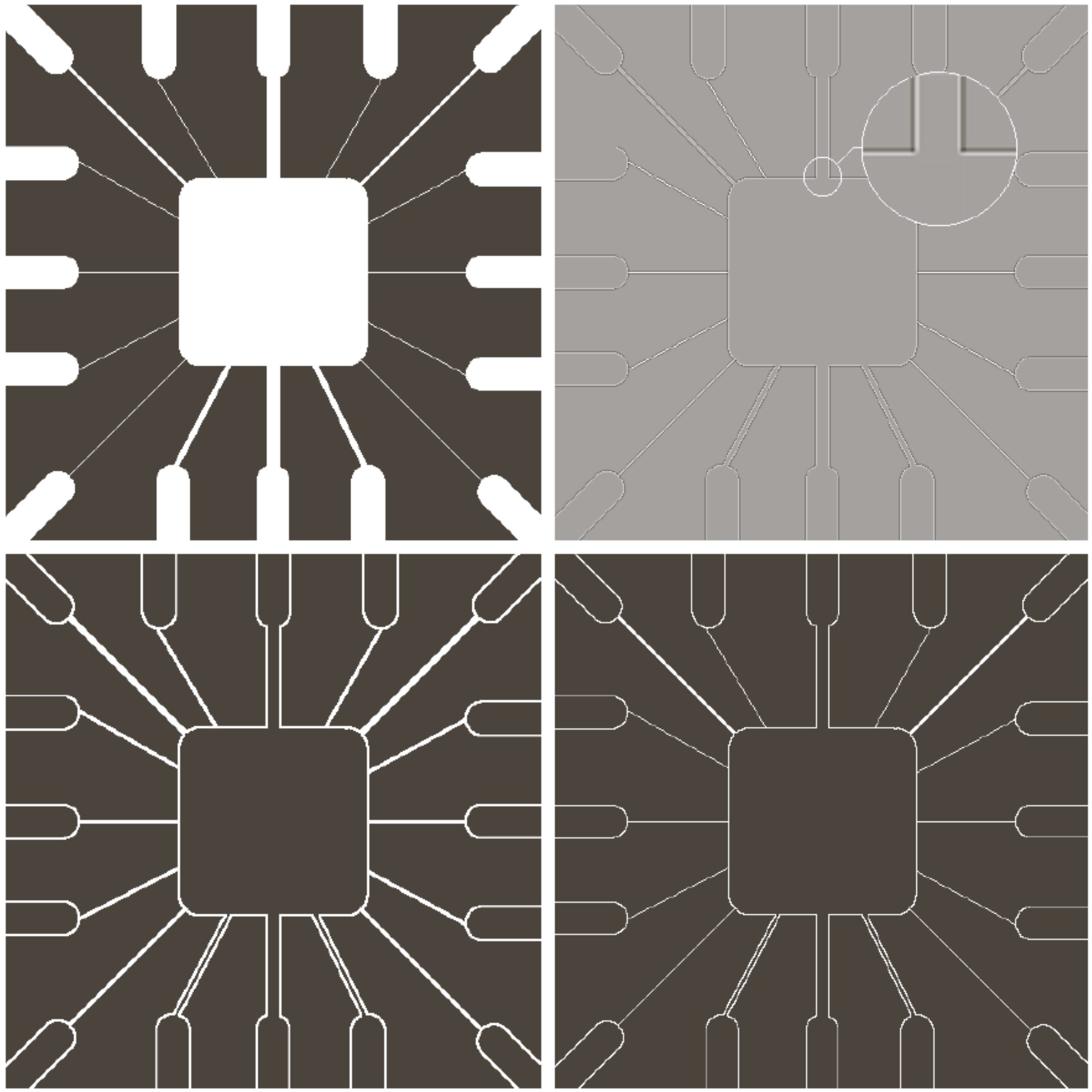
a
b c d

FIGURE 10.4
(a) Point detection (Laplacian) mask. (b) X-ray image of turbine blade with a porosity. The porosity contains a single black pixel. (c) Result of convolving the mask with the image. (d) Result of using Eq. (10.2-8) showing a single point (the point was enlarged to make it easier to see). (Original image courtesy of X-TEK Systems, Ltd.)

Line Detection

- Second derivatives to result in a stronger response and to produce thinner lines than first derivatives

- Double-line effect of the second derivative must be handled properly



a	b
c	d

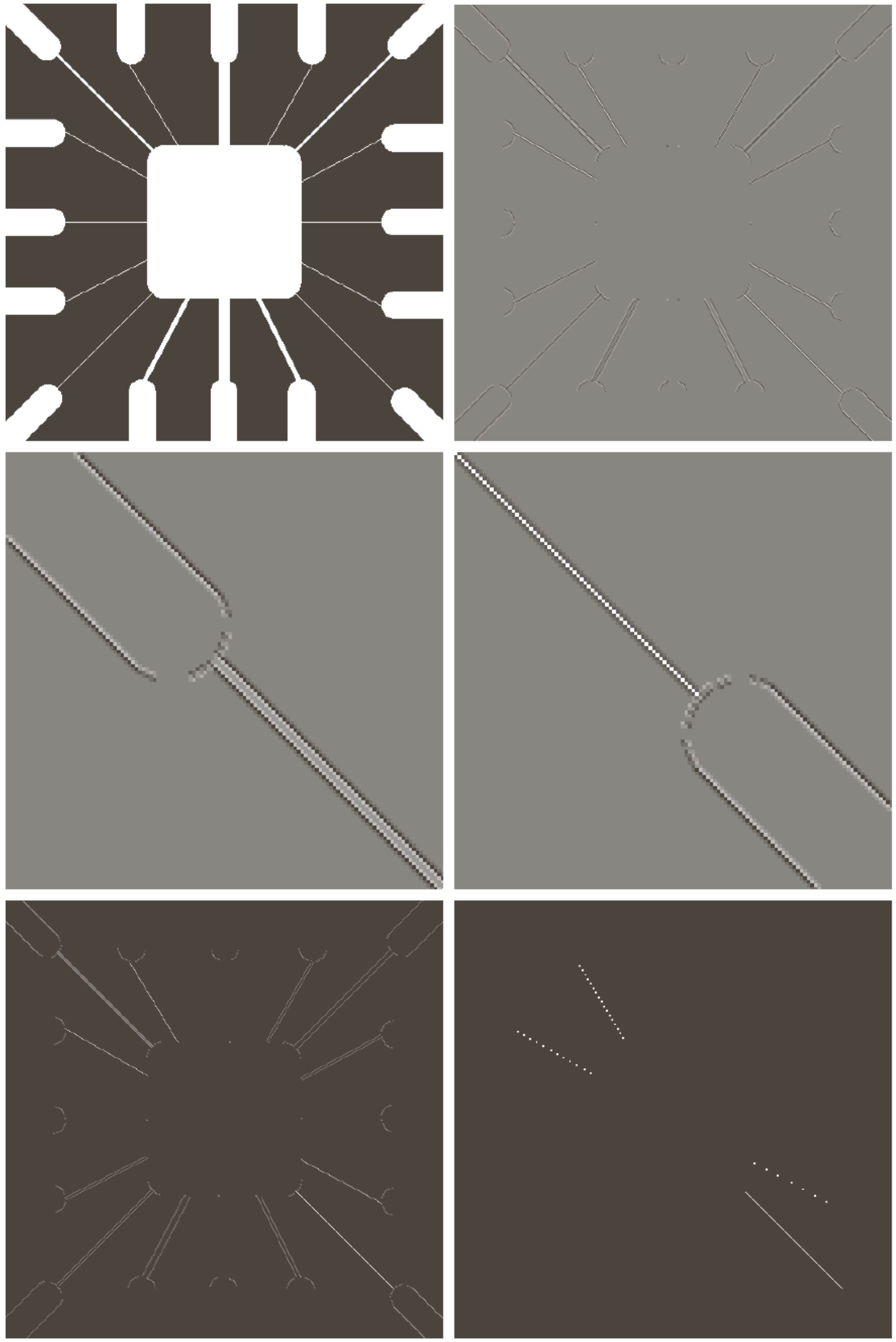
FIGURE 10.5
(a) Original image.
(b) Laplacian image; the magnified section shows the positive/negative double-line effect characteristic of the Laplacian.
(c) Absolute value of the Laplacian.
(d) Positive values of the Laplacian.

Detecting Line in Specified Directions

−1	−1	−1	2	−1	−1	−1	2	−1	−1	−1	2
2	2	2	−1	2	−1	−1	2	−1	−1	2	−1
−1	−1	−1	−1	−1	2	−1	2	−1	2	−1	−1
	Horizontal			+45°			Vertical			−45°	

FIGURE 10.6 Line detection masks. Angles are with respect to the axis system in Fig. 2.18(b).

- Let R_1, R_2, R_3, and R_4 denote the responses of the masks in Fig. 10.6. If, at a given point in the image, $|R_k|>|R_j|$, for all $j \neq k$, that point is said to be more likely associated with a line in the direction of mask k.

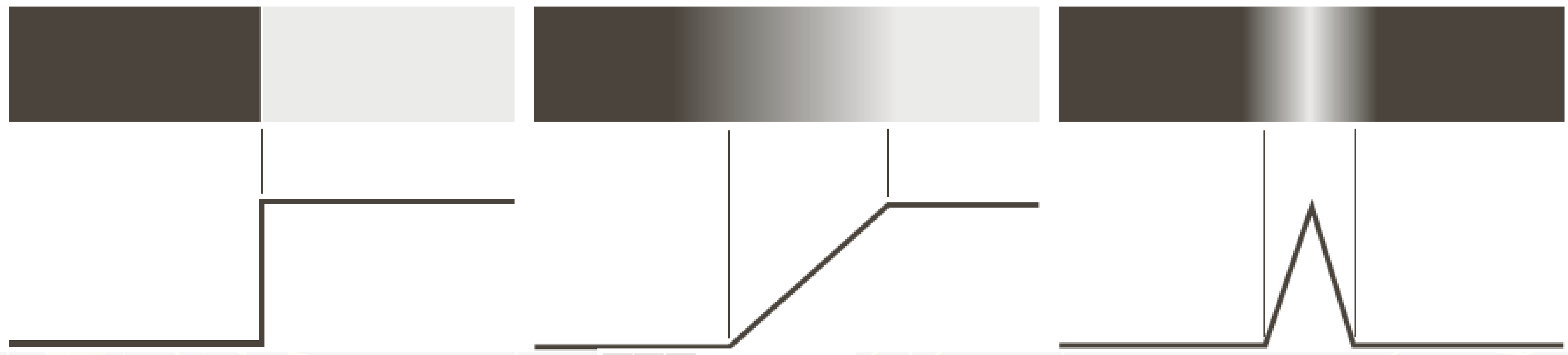


a	b
c	d
e	f

FIGURE 10.7
(a) Image of a wire-bond template.
(b) Result of processing with the +45° line detector mask in Fig. 10.6.
(c) Zoomed view of the top left region of (b).
(d) Zoomed view of the bottom right region of (b). (e) The image in (b) with all negative values set to zero. (f) All points (in white) whose values satisfied the condition $g \geq T$, where g is the image in (e). (The points in (f) were enlarged to make them easier to see.)

Edge Detection

- Edges are those places in an image that correspond to object boundaries.
- Edges are pixels where the brightness function changes abruptly
- Edge models



a b c

FIGURE 10.8 From left to right, models (ideal representations) of a step, a ramp, and a roof edge, and their corresponding intensity profiles.

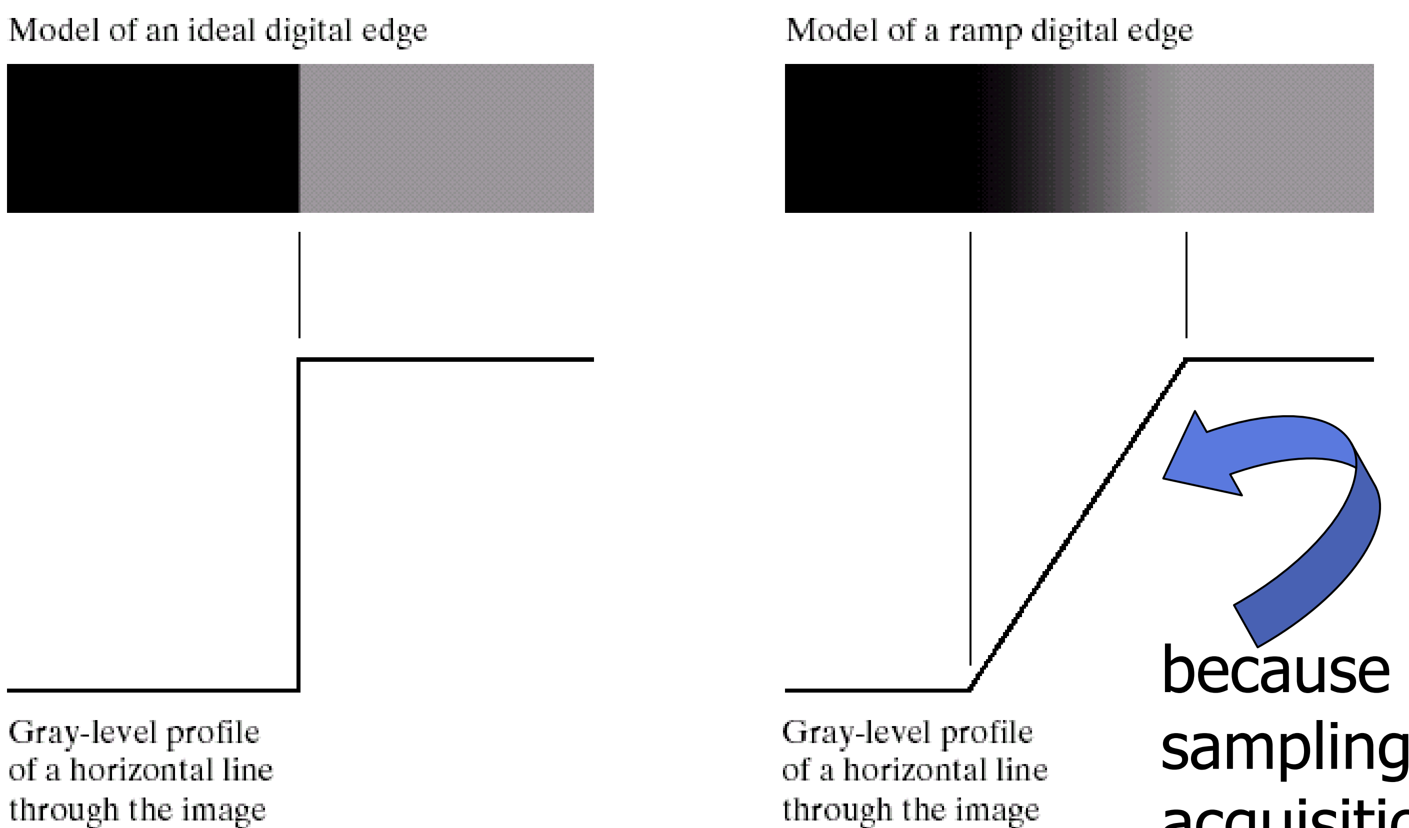
Edge Detection

- Edge information in an image is found by looking at the relationship a pixel has with its neighborhoods.

- If a pixel's gray-level value is similar to those around it, there is probably <u>not</u> an edge at that point.

- If a pixel's has neighbors with widely varying gray levels, it may present an edge point.

Ideal and Ramp Edges



a b

FIGURE 10.5 (a) Model of an ideal digital edge. (b) Model of a ramp edge. The slope of the ramp is proportional to the degree of blurring in the edge.

because of optics, sampling, image acquisition imperfection

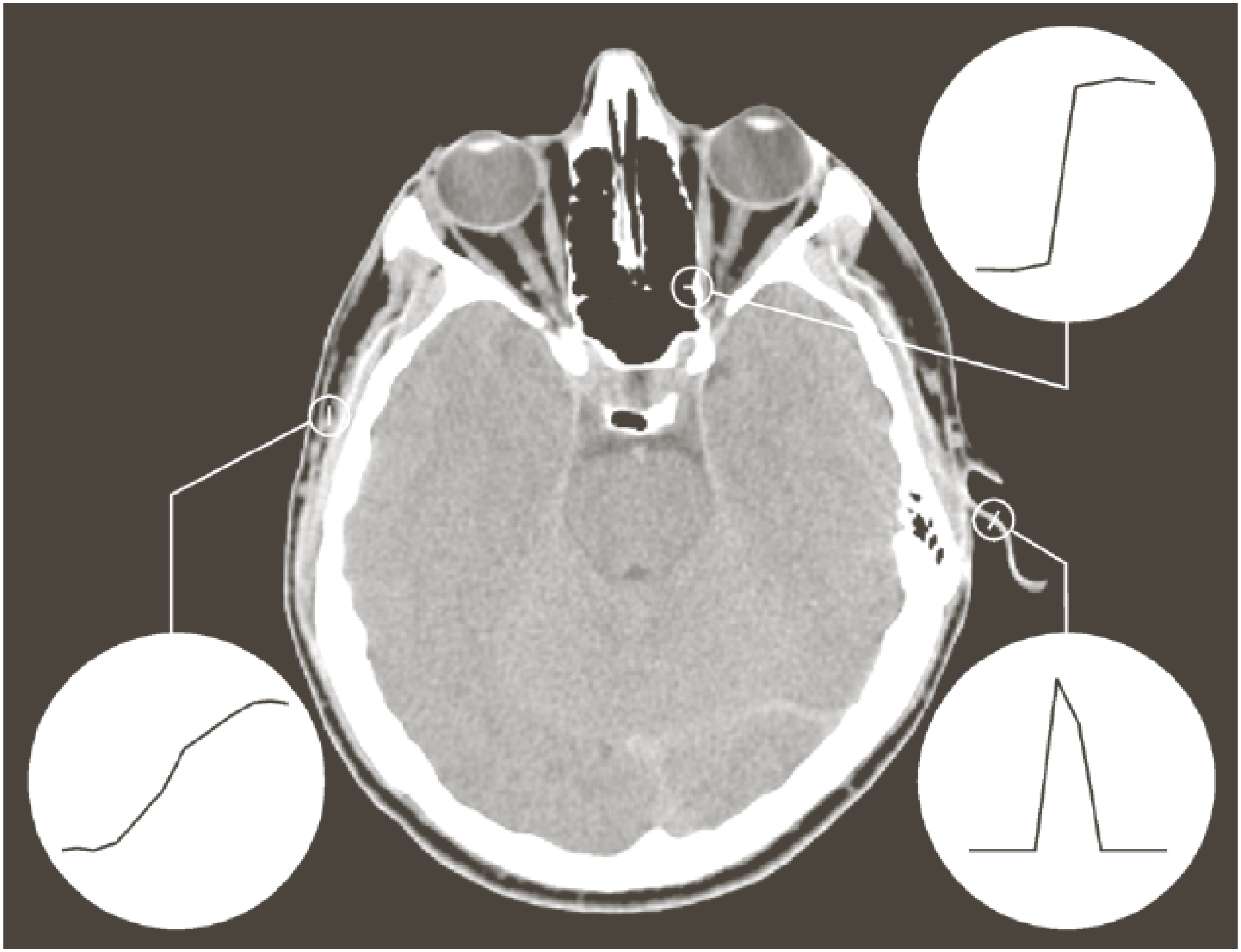
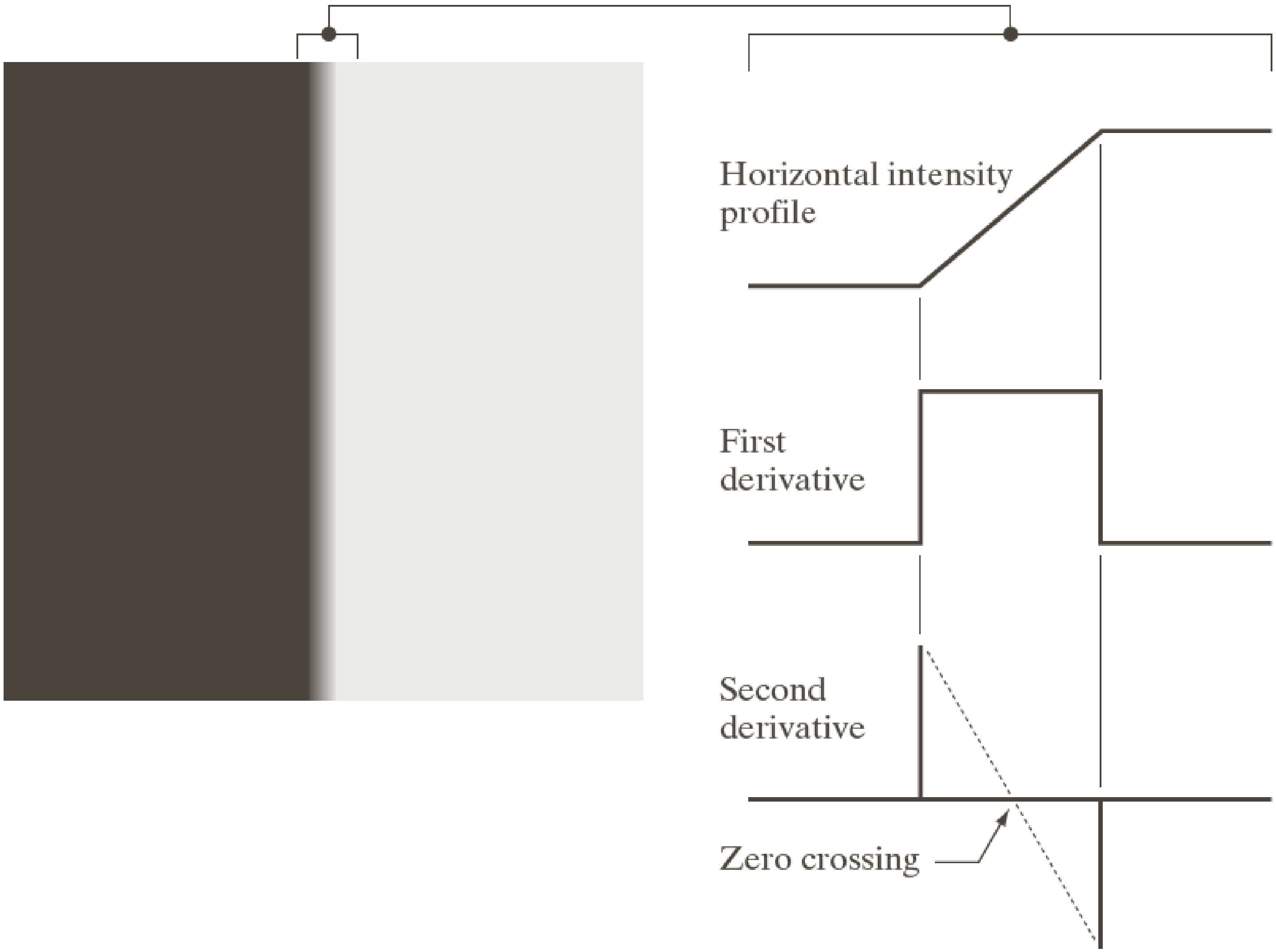


FIGURE 10.9 A 1508 × 1970 image showing (zoomed) actual ramp (bottom, left), step (top, right), and roof edge profiles. The profiles are from dark to light, in the areas indicated by the short line segments shown in the small circles. The ramp and "step" profiles span 9 pixels and 2 pixels, respectively. The base of the roof edge is 3 pixels. (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)



a b

FIGURE 10.10 (a) Two regions of constant intensity separated by an ideal vertical ramp edge. (b) Detail near the edge, showing a horizontal intensity profile, together with its first and second derivatives.

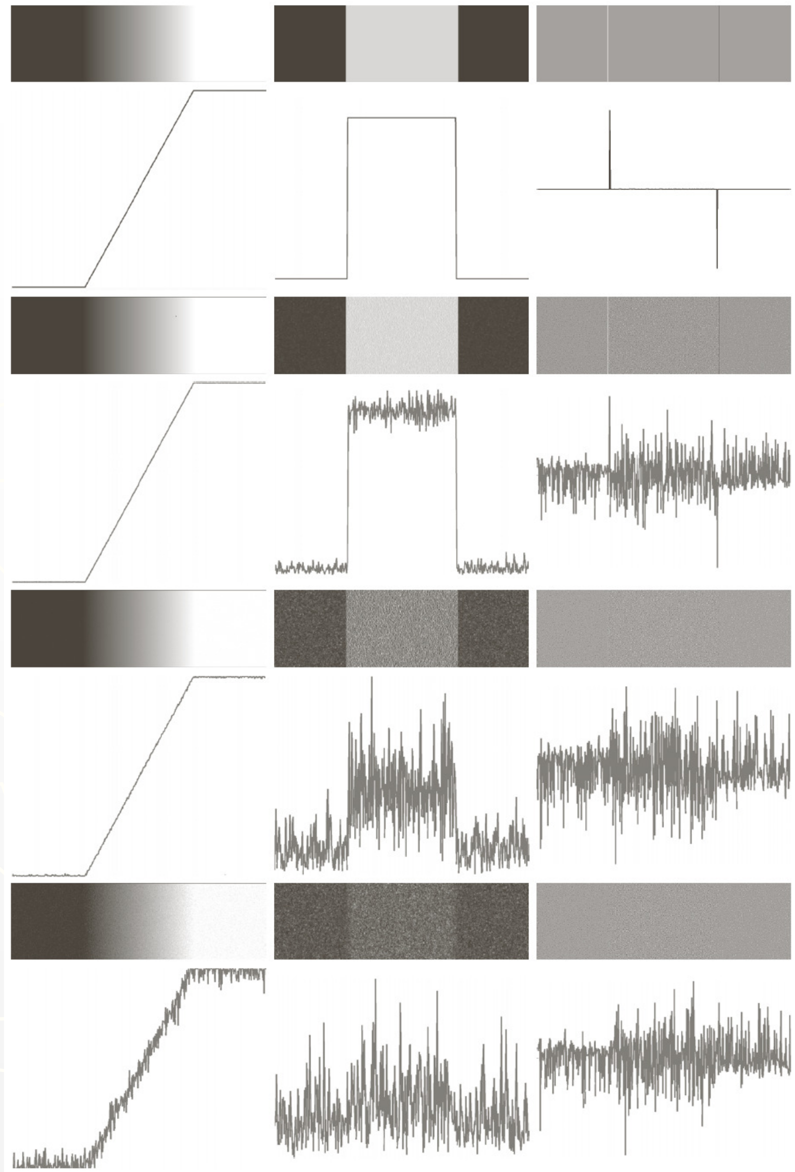


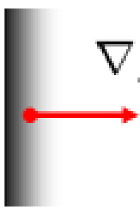
FIGURE 10.11 First column: Images and intensity profiles of a ramp edge corrupted by random Gaussian noise of zero mean and standard deviations of 0.0, 0.1, 1.0, and 10.0 intensity levels, respectively. Second column: First-derivative images and intensity profiles. Third column: Second-derivative images and intensity profiles.

Basic Edge Detection by Using First-Order Derivative

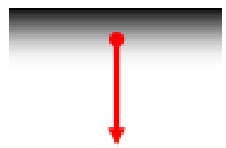
The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

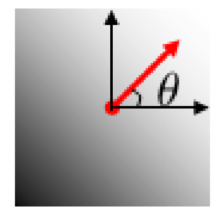
The gradient points in the direction of most rapid change in intensity



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

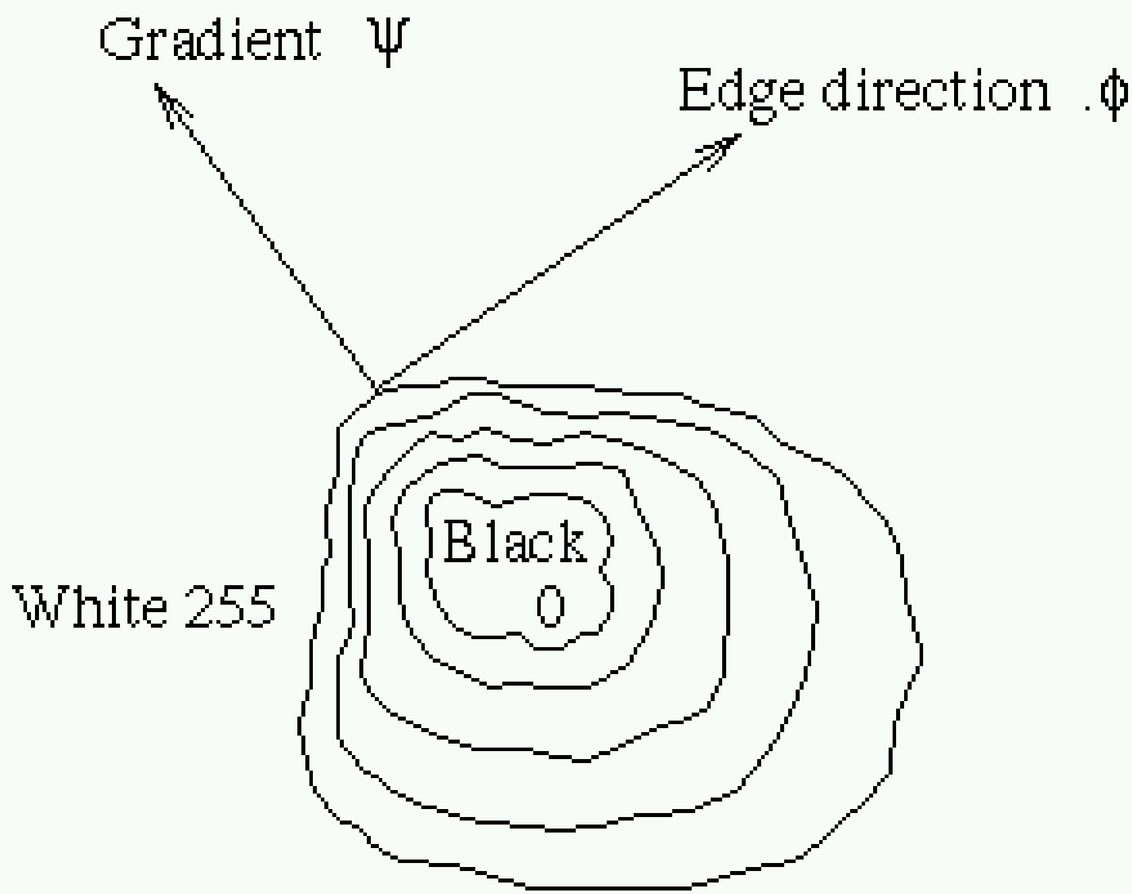
The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

The *edge strength* is given by the gradient magnitude

$$||\nabla f|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$


Gradient ψ
Edge direction .ϕ
Black
0
White 255

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0	0	−1
0	1	1	0

Roberts

−1	−1	−1	−1	0	1
0	0	0	−1	0	1
1	1	1	−1	0	1

Prewitt

−1	−2	−1	−1	0	1
0	0	0	−2	0	2
1	2	1	−1	0	1

Sobel

a
b c
d e
f g

FIGURE 10.14 A 3×3 region of an image (the z's are intensity values) and various masks used to compute the gradient at the point labeled z_5.

0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

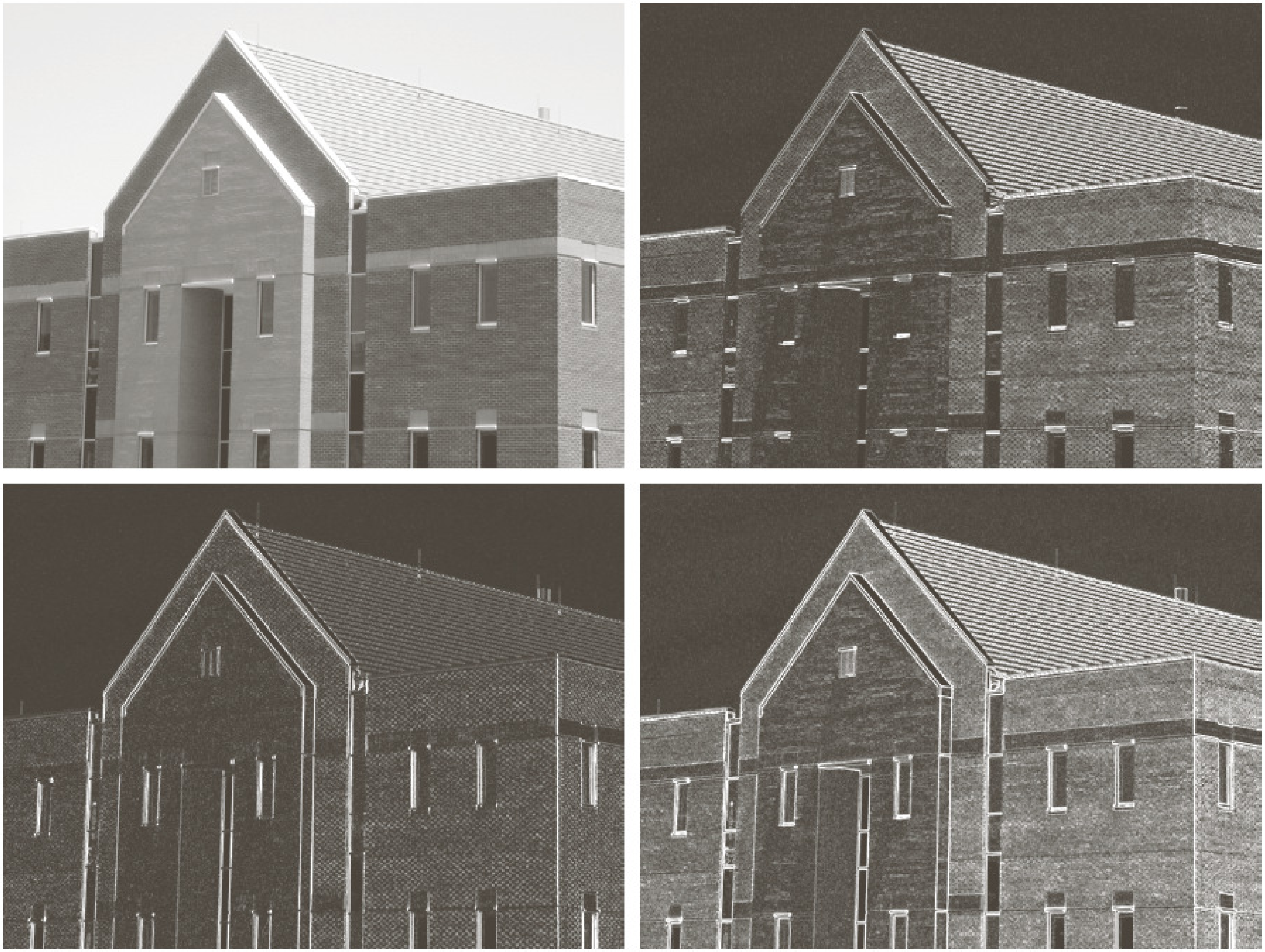
0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Sobel

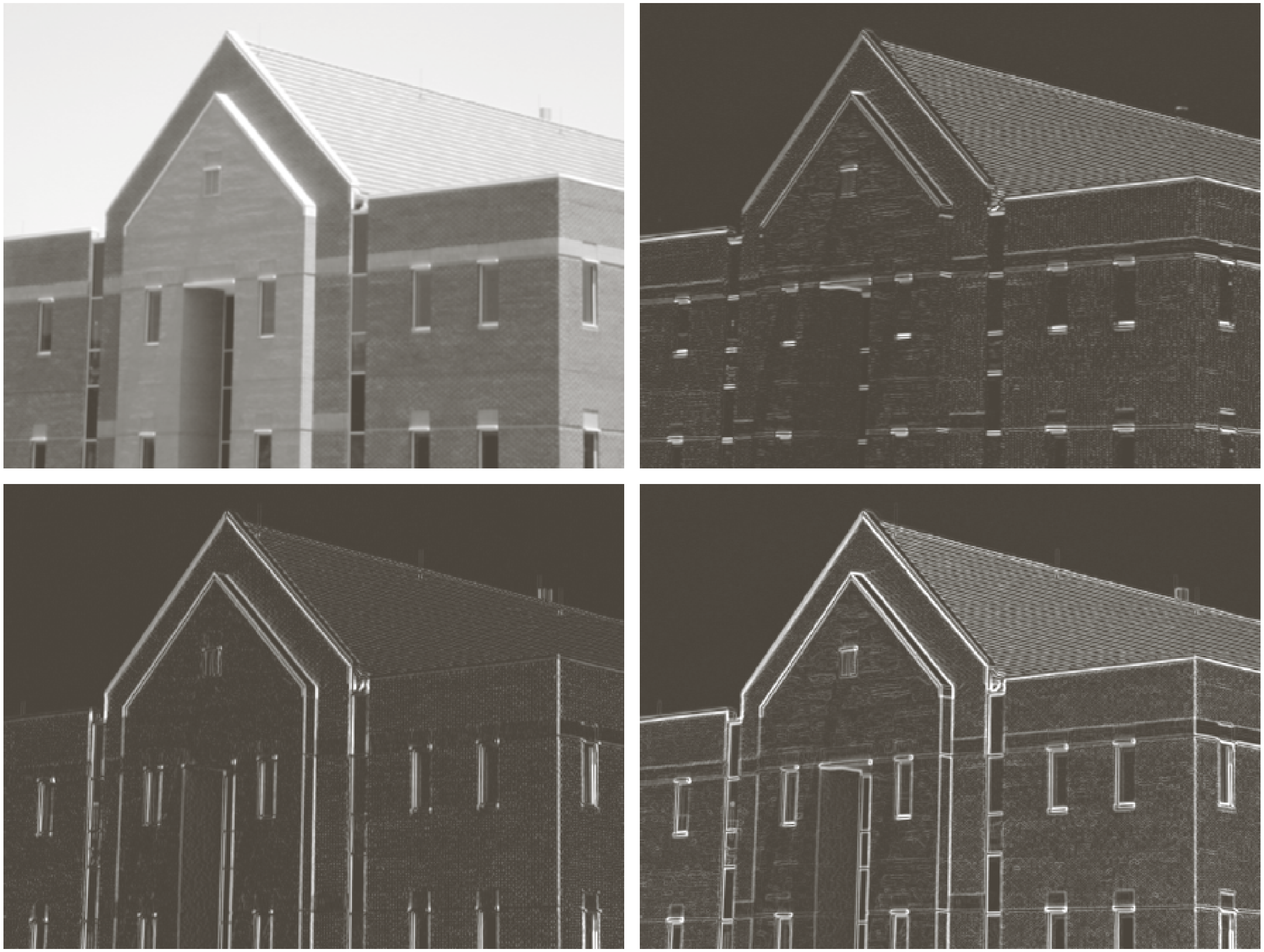
a b
c d

FIGURE 10.15 Prewitt and Sobel masks for detecting diagonal edges.



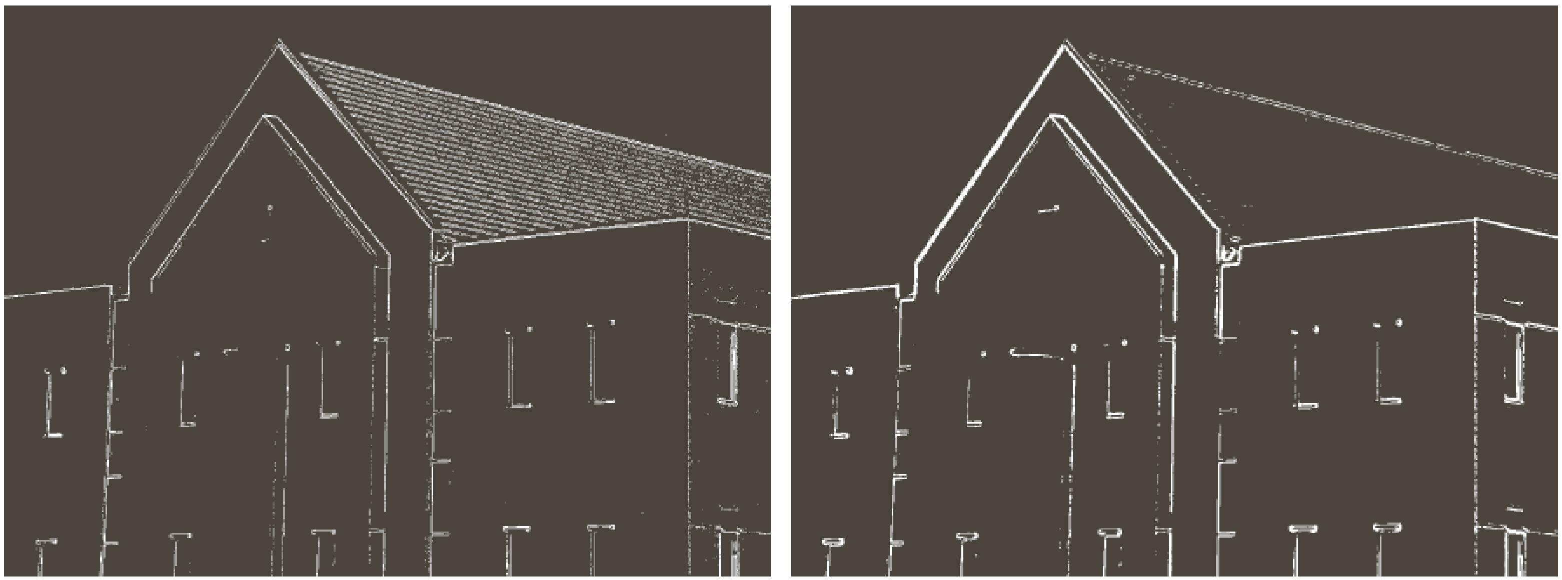
a	b
c	d

FIGURE 10.16
(a) Original image of size 834 × 1114 pixels, with intensity values scaled to the range [0, 1]. (b) $|g_x|$, the component of the gradient in the x-direction, obtained using the Sobel mask in Fig. 10.14(f) to filter the image. (c) $|g_y|$, obtained using the mask in Fig. 10.14(g). (d) The gradient image, $|g_x| + |g_y|$.



a	b
c	d

FIGURE 10.18 Same sequence as in Fig. 10.16, but with the original image smoothed using a 5×5 averaging filter prior to edge detection.



a b

FIGURE 10.20 (a) Thresholded version of the image in Fig. 10.16(d), with the threshold selected as 33% of the highest value in the image; this threshold was just high enough to eliminate most of the brick edges in the gradient image. (b) Thresholded version of the image in Fig. 10.18(d), obtained using a threshold equal to 33% of the highest value in that image.

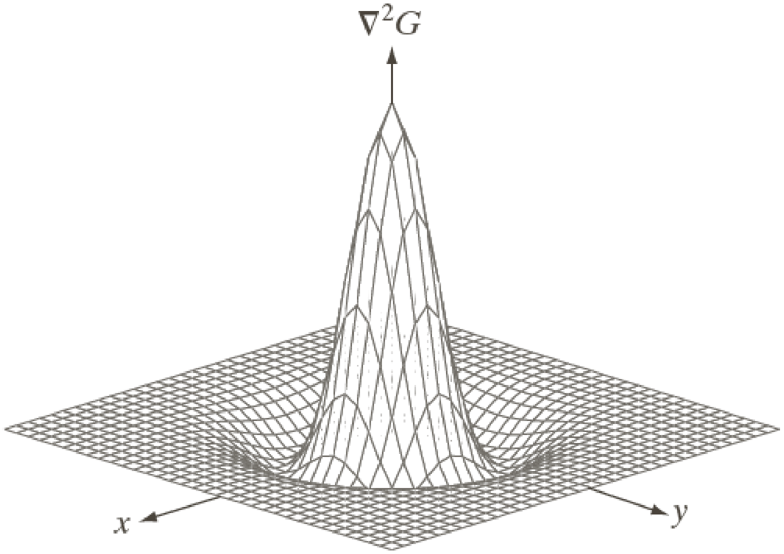
Advanced Techniques for Edge Detection

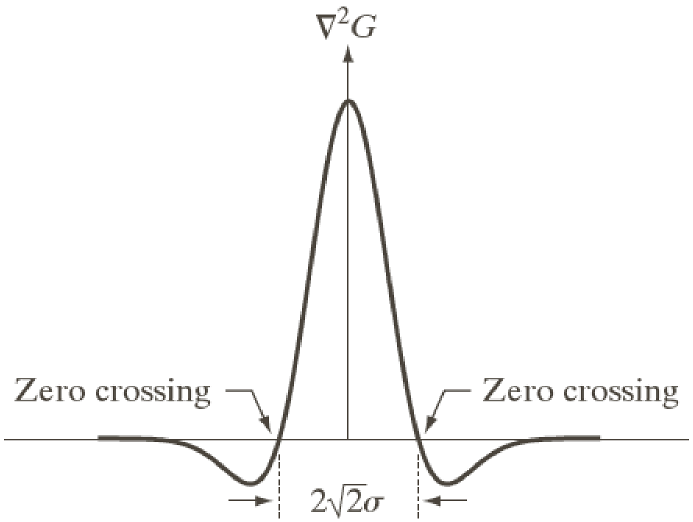
- The Marr-Hildreth edge detector

$$G(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}, \ \sigma : \text{space constant.}$$

Laplacian of Gaussian (LoG)

$$\nabla^2 G(x, y) = \frac{\partial^2 G(x, y)}{\partial x^2} + \frac{\partial^2 G(x, y)}{\partial y^2}$$





0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
0	0	−1	0	0

a b
c d

FIGURE 10.21
(a) Three-dimensional plot of the *negative* of the LoG. (b) Negative of the LoG displayed as an image. (c) Cross section of (a) showing zero crossings.
(d) 5 × 5 mask approximation to the shape in (a). The negative of this mask would be used in practice.

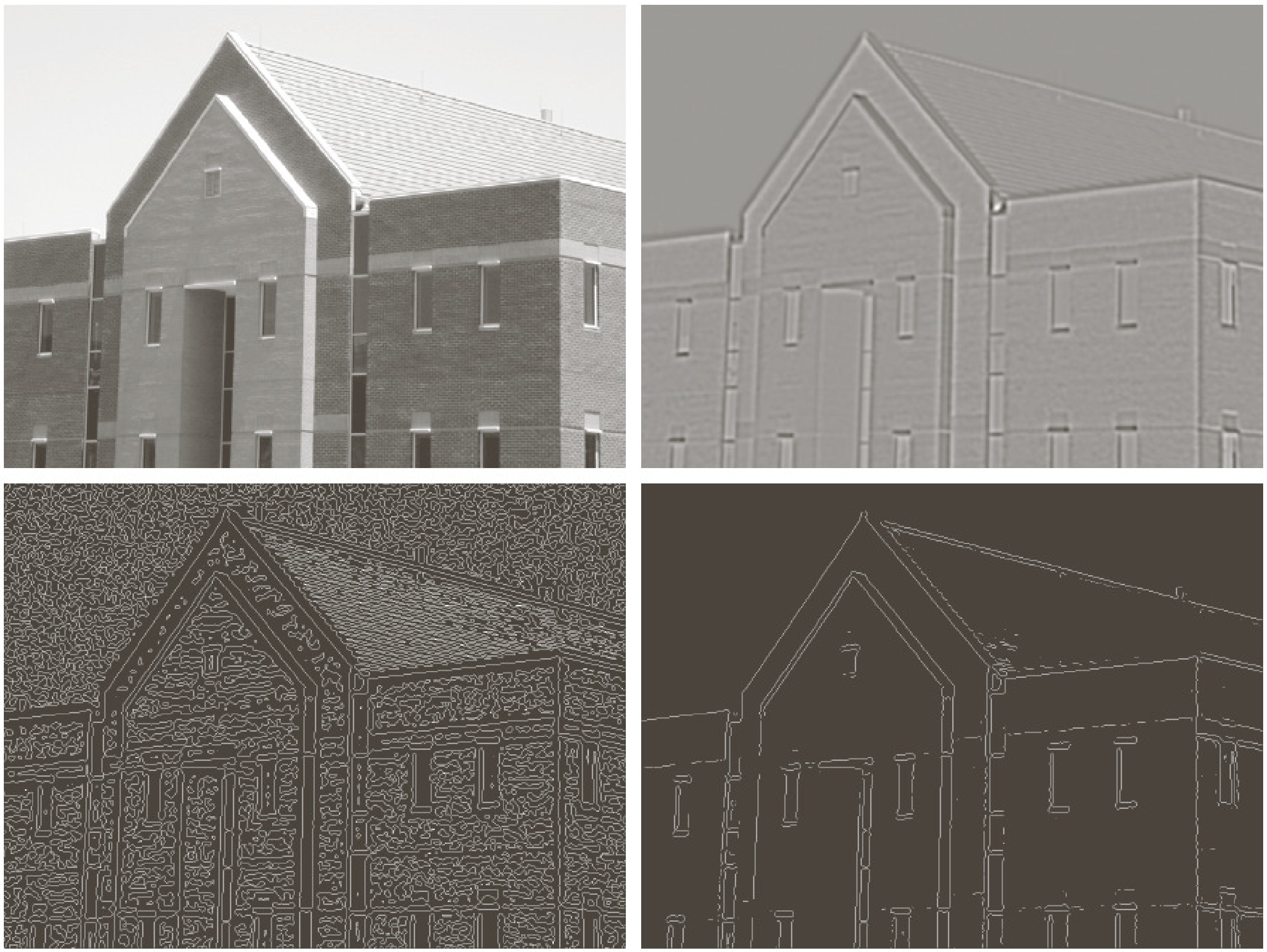
Marr-Hildreth Algorithm

1. Filter the input image with an nxn Gaussian lowpass filter. N is the smallest odd integer greater than or equal to 6σ

2. Compute the Laplacian of the image resulting from step1

3. Find the zero crossing of the image from step 2

$$g(x, y) = \nabla^2 \left[G(x, y) \star f(x, y) \right]$$



a b
c d

FIGURE 10.22
(a) Original image of size 834 × 1114 pixels, with intensity values scaled to the range [0, 1]. (b) Results of Steps 1 and 2 of the Marr-Hildreth algorithm using $\sigma = 4$ and $n = 25$. (c) Zero crossings of (b) using a threshold of 0 (note the closed-loop edges). (d) Zero crossings found using a threshold equal to 4% of the maximum value of the image in (b). Note the thin edges.

The Canny Edge Detector

- **Optimal for step edges corrupted by white noise.**

- **The Objective**

1. **Low error rate**
   The edges detected must be as close as possible to the true edge

2. **Edge points should be well localized**
   The edges located must be as close as possible to the true edges

3. **Single edge point response**
   The number of local maxima around the true edge should be minimum

The Canny Edge Detector: Algorithm (1)

Let $f(x, y)$ denote the input image and $G(x, y)$ denote the Gaussian function:

$$G(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

We form a smoothed image, $f_s(x, y)$ by convolving G and f:

$$f_s(x, y) = G(x, y) \bigstar f(x, y)$$

The Canny Edge Detector: Algorithm(2)

Compute the gradient magnitude and direction (angle):

$$M(x, y) = \sqrt{g_x^{\,2} + g_y^{\,2}}$$

and

$$\alpha(x, y) = \arctan(g_y / g_x)$$

where $g_x = \partial f_s / \partial x$ and $g_y = \partial f_s / \partial y$

Note: any of the filter mask pairs in Fig.10.14 can be used to obtain g_x and g_y

The Canny Edge Detector: Algorithm(3)

The gradient $M(x, y)$ typically contains wide ridge around local maxima. Next step is to thin those ridges.

<u>Nonmaxima suppression:</u>

Let d_1, d_2, d_3, and d_4 denote the four basic edge directions for a 3×3 region: horizontal, -45°, vertical, +45°, respectively.

1. Find the direction d_k that is closest to $\alpha(x, y)$.
2. If the value of $M(x, y)$ is less than at least one of its two neighbors along d_k, let $g_N(x, y) = 0$ (suppression); otherwise, let $g_N(x, y) = M(x, y)$

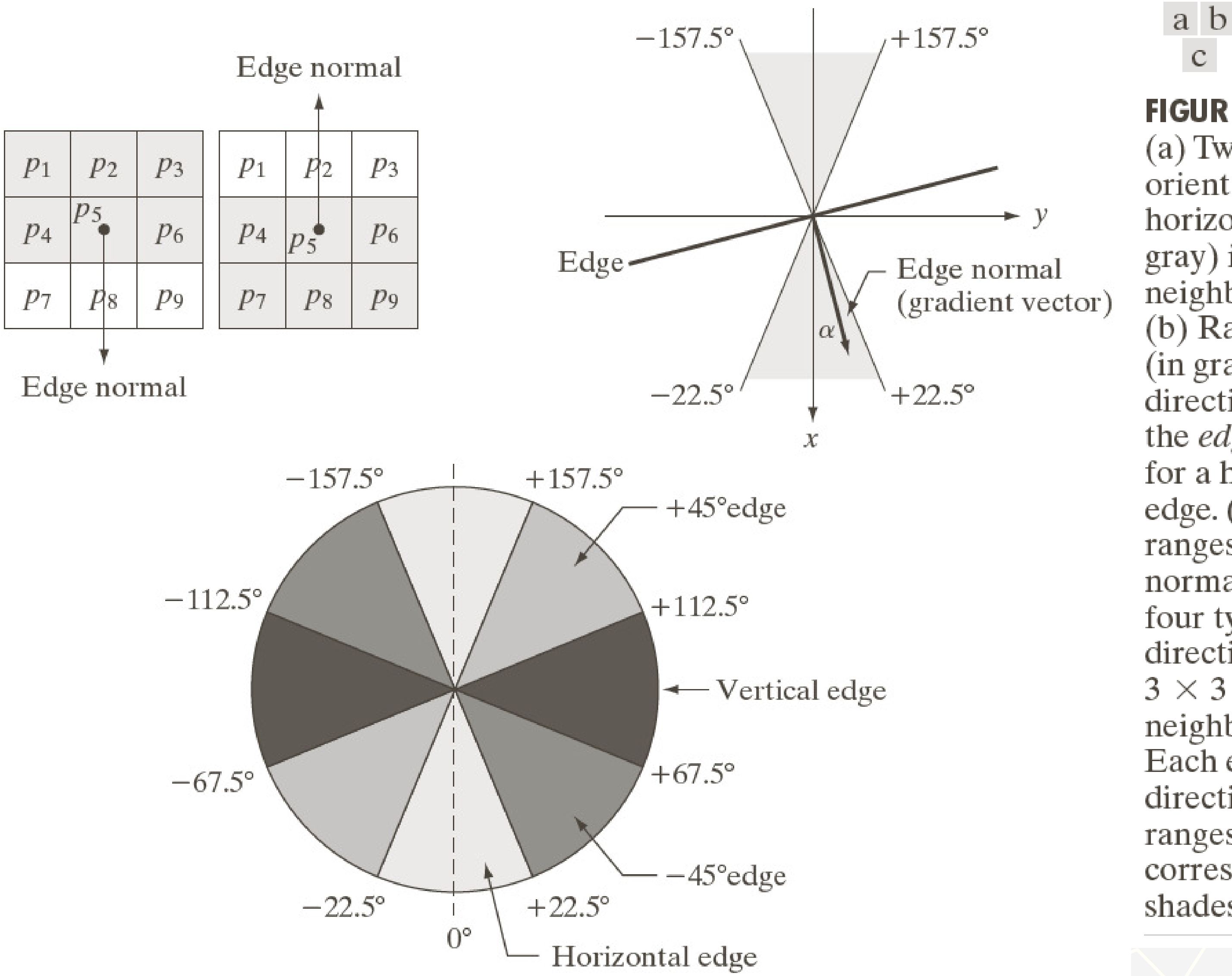


FIGURE 10.24
(a) Two possible orientations of a horizontal edge (in gray) in a 3×3 neighborhood. (b) Range of values (in gray) of α, the direction angle of the *edge normal*, for a horizontal edge. (c) The angle ranges of the edge normals for the four types of edge directions in a 3×3 neighborhood. Each edge direction has two ranges, shown in corresponding shades of gray.

The Canny Edge Detector: Algorithm(4)

The final operation is to threshold $g_N(x, y)$ to reduce false edge points.

Hysteresis thresholding:

$$g_{NH}(x, y) = g_N(x, y) \geq T_H$$

$$g_{NL}(x, y) = g_N(x, y) \geq T_L$$

and

$$g_{NL}(x, y) = g_{NL}(x, y) - g_{NH}(x, y)$$

The Canny Edge Detector: Algorithm(5)

Depending on the value of T_H, the edges in $g_{NH}(x, y)$ typically have gaps. Longer edges are formed using the following procedure:

(a). Locate the next unvisited edge pixel, p, in $g_{NH}(x, y)$.

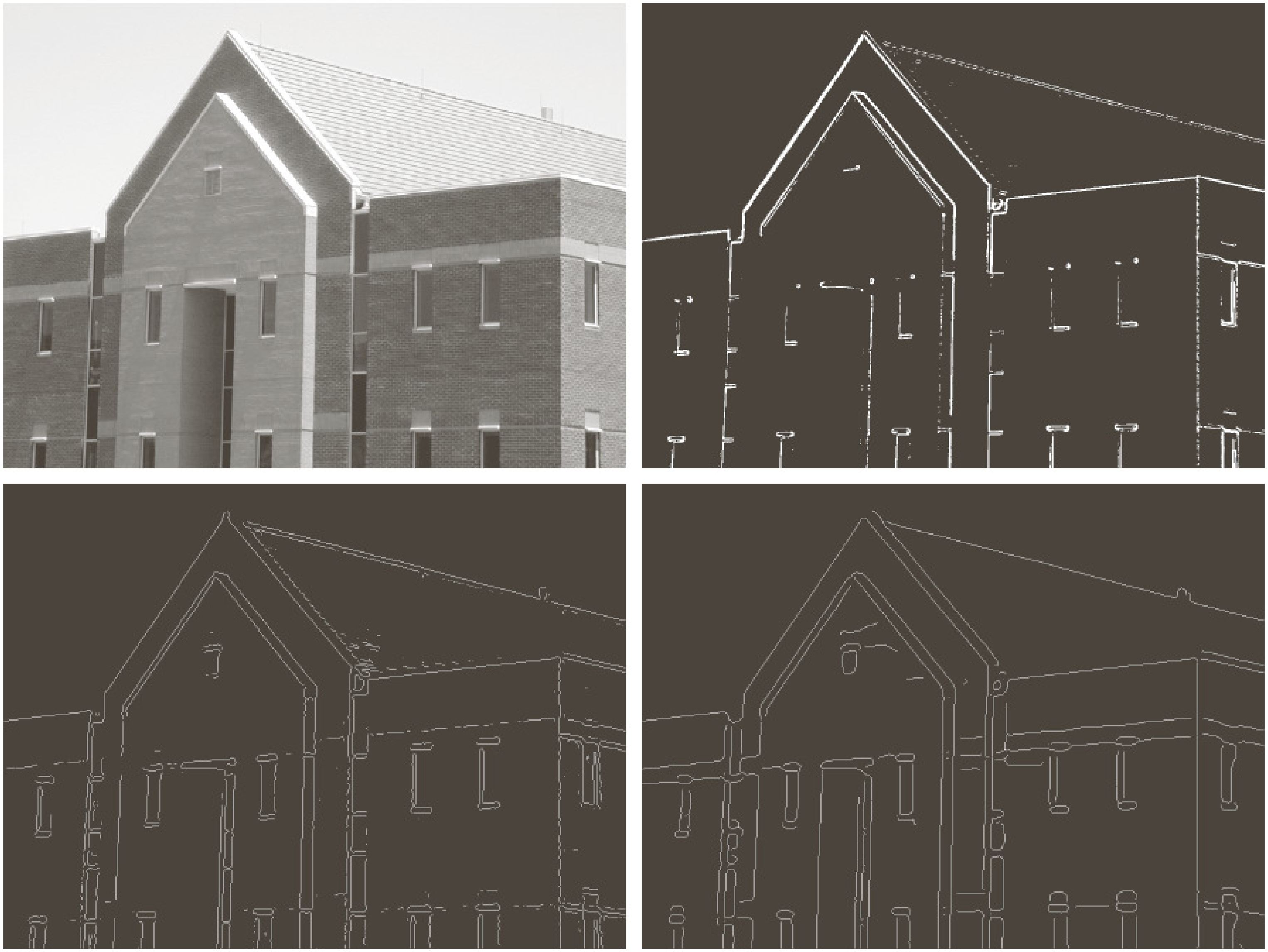
(b). Mark as valid edge pixel all the weak pixels in $g_{NL}(x, y)$ that are connected to p using 8-connectivity.

(c). If all nonzero pixel in $g_{NH}(x, y)$ have been visited go to step (d), esle return to (a).

(d). Set to zero all pixels in $g_{NL}(x, y)$ that were not marked as valid edge pixels.

The Canny Edge Detection: Summary

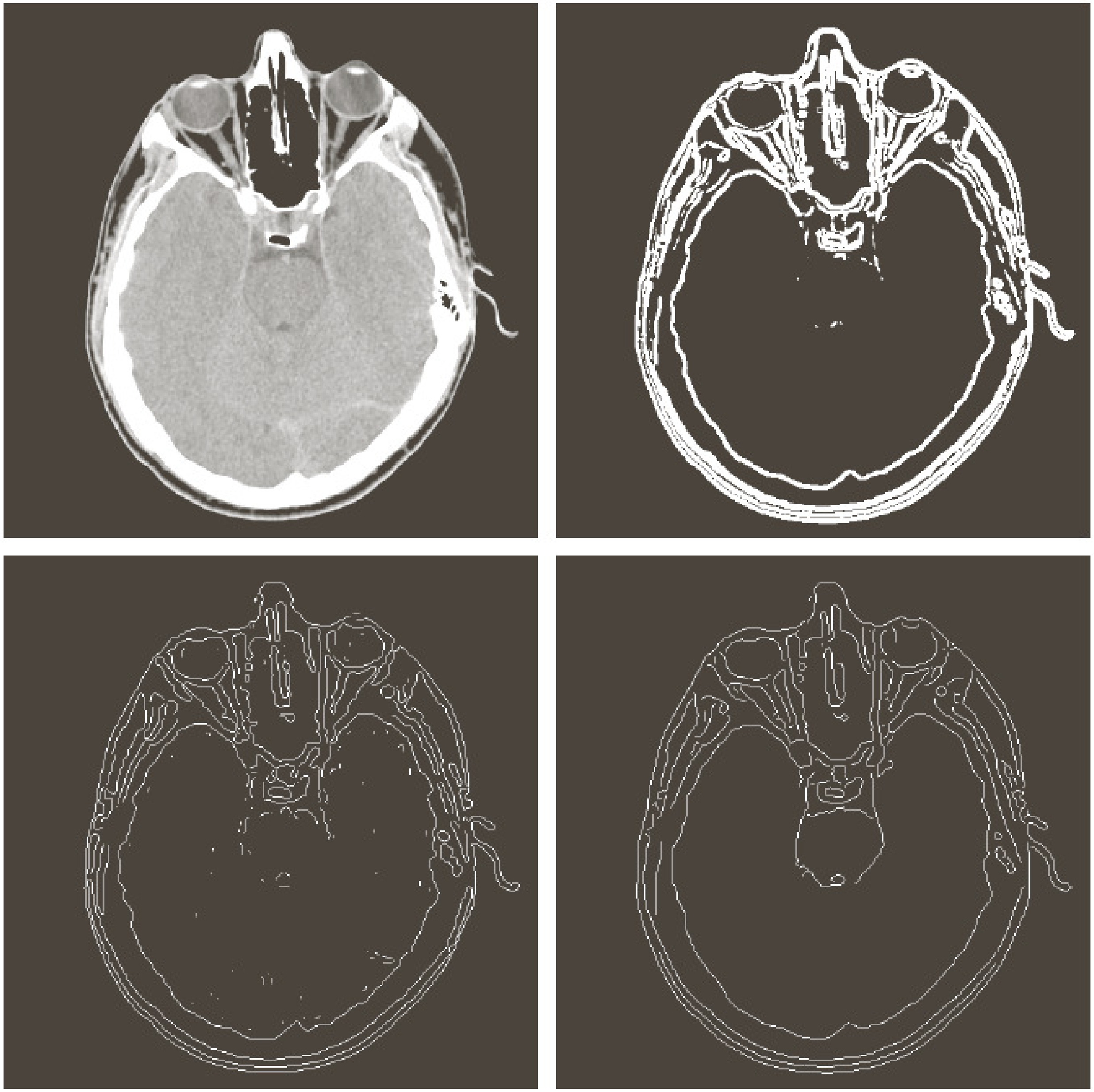
- Smooth the input image with a Gaussian filter
- Compute the gradient magnitude and angle images
- Apply nonmaxima suppression to the gradient magnitude image
- Use double thresholding and connectivity analysis to detect and link edges



a	b
c	d

FIGURE 10.25 (a) Original image of size 834 × 1114 pixels, with intensity values scaled to the range [0, 1]. (b) Thresholded gradient of smoothed image. (c) Image obtained using the Marr-Hildreth algorithm. (d) Image obtained using the Canny algorithm. Note the significant improvement of the Canny image compared to the other two.

$T_L = 0.04; T_H = 0.10; \sigma = 4$ and a mask of size 25×25



a b
c d

FIGURE 10.26
(a) Original head CT image of size 512 × 512 pixels, with intensity values scaled to the range [0, 1]. (b) Thresholded gradient of smoothed image. (c) Image obtained using the Marr-Hildreth algorithm. (d) Image obtained using the Canny algorithm. (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

$T_L = 0.05; T_H = 0.15; \sigma = 2$ and a mask of size 13×13

Edge Linking and Boundary Detection

- Edge detection typically is followed by linking algorithms designed to assemble edge pixels into meaningful edges and/or region boundaries

Local Processing

- Analyze the characteristics of pixels in a small neighborhood about every point (x,y) that has been declared an edge point

- All points that similar according to predefined criteria are linked, forming an edge of pixels.

Establishing similarity: (1) the strength (magnitude) and (2) the direction of the gradient vector.

A pixel with coordinates (s,t) in S_{xy} is linked to the pixel at (x,y) if both magnitude and direction criteria are satisfied.

Local Processing

Let S_{xy} denote the set of coordinates of a neighborhood centered at point (x, y) in an image. An edge pixel with coordinate (s,t) in S_{xy} is similar in *magnitude* to the pixel at (x, y) if

$$\left|M(s,t)-M(x,y)\right| \le E$$

An edge pixel with coordinate (s,t) in S_{xy} is similar in *angle* to the pixel at (x, y) if

$$\left|\alpha(s,t)-\alpha(x,y)\right| \le A$$